

Exercise 7

Find the closed form function for the following Taylor series:

$$f(x) = 1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6 + \dots$$

Solution

$$\begin{aligned} f(x) &= 1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6 + \dots \\ f(x) &= \frac{(2x)^0}{0!} + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!} + \dots \\ f(x) &= \sum_{n=0}^{\infty} \frac{1}{(2n)!} (2x)^{2n} \end{aligned}$$

Therefore,

$$f(x) = \cosh 2x.$$